Scattering of high energy electrons from oriented nuclei in eikonal approximation

1974 J. Phys. A: Math. Nucl. Gen. 7 2304

(http://iopscience.iop.org/0301-0015/7/18/011)

J. Phys. A: Math., Nucl. Gen., Vol. 7, No. 18, 1974. Printed in Great Britain. © 1974

Scattering of high energy electrons from oriented nuclei in eikonal approximation

S Sarkar and Sofia Khatun

Department of Theoretical Physics, Indian Association for the Cultivation of Science, Jadavpur, Calcutta 32, India

Received 10 May 1974, in final form 9 September 1974

Abstract. Using the eikonal approximation of Yennie we give a general expression for the cross section for fast-electron scattering from oriented nuclei and a corresponding simpler expression for nuclei fully oriented along the direction of momentum transfer. The branching ratios of quadrupole transition probabilities to different rotational levels of aligned nuclei are also obtained. Using Petkov's analytic method for the Fermi charge density and taking different forms of nuclear charge distributions we calculate in the eikonal approximation the cross section and the effect of nuclear alignment on it for electron scattering from oriented ^{165}Ho nuclei. We compare our results with those obtained by Wright employing phase-shifted distorted waves and the experimental findings of Safrata. The Born approximation limit of the above results is also obtained. A correction term is added to Petkov's result for the quadrupole transition probability. In the case when one assumes a δ function quadrupole distribution the effect of nuclear alignment, determined in the eikonal approximation, is compared with that obtained by Greenstein in the Schiff–Tiemann approximation. We further investigate the variation of the above mentioned effect with the change in energy and sign of the charge of the projectile, the degree and direction of nuclear alignment

1. Introduction

In this paper we give a general formulation for the study of scattering of high energy electrons from an oriented deformed nucleus in the eikonal approximation due to Yennie (1965, to be referred to as paper I). This approximation takes into account the Coulomb distortion of the electron wave in the field of the medium and heavy nuclei. The same problem (for ^{165}Ho) was treated by Wright (1969) who used the method of phase-shifted distorted waves (to be referred to as Wright's method) obtained by solving the Dirac equation with the spherical part of the nuclear charge distribution only. In this method, which is very tedious and involves great difficulties when applied to large-angle scattering, the explicit relation between the charge distribution and the structure of the scattering cross section is lost. However, Yennie's method leads to a form which exhibits this explicit relation clearly. This method consists of treating the electron–nucleus scattering problem in perturbation theory taking the distorted eikonal wavefunctions as zero-order states. For large-angle scattering which is particularly required in determining the nuclear structure Yennie's method gives a result which is in good agreement with that obtained by Wright's method. Furthermore it saves much computing time. In the work of Wallace (1973) we find another approach to the eikonal approximation for the potential scattering problem which is based on T matrix formalism. He made some improvements on Glauber's (1959) work regarding the eikonal approximation (which is limited in its validity to small scattering angles), for extending the domain of validity of the eikonal approximation to large scattering angles. For a potential which produces oscillatory or diffraction-like differential scattering, Wallace's theory is not very satisfactory in the domain of large-angle scattering. His theory (formulated for a spherically symmetric potential) is thus not suitable for dealing with large-angle scattering of electrons from non-spherical nuclei.

Using the Bohr–Mottelson model to describe the low-lying rotational levels of a nucleus we obtain, in the eikonal approximation, an expression for the cross section for the scattering of electrons from aligned nuclei in a form which involves Clebsch–Gordan coefficients, Racah coefficients, and the statistical tensor of Fano f_I (describing the degree of orientation of a nucleus) in the same way as found in the corresponding Born approximation result previously derived by Inopin and Tishchenko (1960). We also find that this cross section assumes a simpler form which involves Clebsch–Gordan coefficients and f_I's but no Racah coefficients when we take the nucleus to be fully oriented along the direction of momentum transfer. We note that at the present state of the experimental situation it is not possible to separate the inelastic scattering associated with the transition (which is related to the quadrupole charge distribution) to the low-lying rotational levels of a heavy nucleus (such as ^{165}Ho). We take into account the contribution of these processes to the scattering cross section. In some cases (to be stated later) the contribution of the quadrupole charge distribution to the cross section can be separated out by performing experiments with nuclei aligned in some particular ways.

Inopin and Tishchenko (1960) obtained the branching ratio of the inelastic cross sections involving different excited levels of the same rotational band of a non-aligned nucleus in the Born approximation. The same result for the branching ratio can be obtained in the eikonal approximation. Here we also calculate the branching ratios for the partially oriented nuclei and for nuclei fully oriented along the direction of the momentum transfer. The branching ratios may be measured experimentally in the case of light nuclei for which the separation between excited levels is about 1 MeV.

In order to have a good fit with the experimental data different forms for the nuclear charge distribution can be tried theoretically. For ^{88}Sr Onley *et al* (1964) and Petkov *et al* (1967b, to be referred to as II) used two different forms (to be referred to as form 1 and form 2 respectively, which are given in equations (41) and (42)) for the quadrupole transition charge density $\rho_2(r)$, based on different theoretical reasonings. The first form for $\rho_2(r)$ was also assumed by Wright (1969) for the ^{165}Ho nucleus.

In this paper we apply the analytic method of Petkov *et al* (1967a, to be referred to as III), which enables us to avoid numerical integration, to study electron scattering from a nucleus whose spherical part of the charge distribution is Fermi-like and whose quadrupole part of the charge distribution ($\rho_2(r)$) is given by any of the two forms 1 and 2. A correction term is added to the result of Petkov *et al* (1967b) for the inelastic quadrupole transition amplitude where the form 2 for $\rho_2(r)$ is assumed. This enables us to obtain the correct Born approximation result. It is possible to obtain a closed analytic expression for the quadrupole transition amplitude in the Born approximation limit if one uses form 1 (assumed by Wright 1969) instead of form 2 (given by Petkov *et al* 1967b) for $\rho_2(r)$. We find that the algebraic expression for 'c' (related to the nuclear charge distribution) derived here differs from that used in III.

Using the method of eikonal approximation with both forms 1 and 2 for the charge distribution we give numerical results for the scattering cross section for electrons colliding with a randomly-oriented ^{165}Ho nucleus $(\mathrm{d}\sigma/\mathrm{d}\Omega)^{\mathrm{r.n}}$ and also results for Δ (a measure of the alignment effect to be defined later) and compare them with those evaluated by Wright (1969) and the experimental results of Safrata *et al* (1967). For a δ function quadrupole distribution for ^{165}Ho we compare the result (obtained in eikonal approximation) for Δ with that evaluated by Greenstein (1966) in the Schiff–Tiemann approximation. Variations of Δ with changes in the degree of nuclear alignment, the projectile energy and the sign of the charge of the incident particle are also investigated in this paper. We further study how Δ changes with the scattering angle for different directions of orientations. In this connection we may mention that the orientation effects for light nuclei were investigated by Langworthy and Überall (1970) in the Born approximation.

2. Theory

In the eikonal approximation the distorted electron wave is taken in the form given below

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) = \phi_{\boldsymbol{k}}(\boldsymbol{r})\vartheta(\boldsymbol{k}) \exp(\mathrm{i}S(\boldsymbol{r})) \tag{1}$$

where $\phi_{\boldsymbol{k}}(\boldsymbol{r})$ and $S(\boldsymbol{r})$ are the amplitude and the phase of the electron wave respectively, $\vartheta(\boldsymbol{k})$ is the electron spinor.

The scattering amplitude for an electron–nucleus collision as given in I is of the form

$$f = \left(\frac{k}{2\pi}\right) \int \psi_{\boldsymbol{k}_2}^{(-)\dagger}(\boldsymbol{r}) V(\boldsymbol{r}) \psi_{\boldsymbol{k}_1}^{(+)}(\boldsymbol{r})\, \mathrm{d}^3 r \tag{2}$$

where the potential $V(\boldsymbol{r})$ due to the nuclear charge density $\rho(\boldsymbol{r})$ is

$$V(\boldsymbol{r}) = e^2 \int \rho(\boldsymbol{r}')\, \mathrm{d}^3 r' / |\boldsymbol{r}' - \boldsymbol{r}|. \tag{3}$$

The multipole expansion of $\rho(\boldsymbol{r})$ is

$$\rho(\boldsymbol{r}) = \sum_L \rho_L(r) P_L(\hat{\boldsymbol{r}} \cdot \hat{\boldsymbol{N}}) \tag{4}$$

where $\hat{\boldsymbol{N}}$ is the unit vector along the direction of the nuclear symmetry axis.

The potential $V'(r)$ due to $\rho_0(r)$ which is the spherical part of $\rho(\boldsymbol{r})$ is expanded (as in I) as follows

$$V'(r) = V'(0) + \tfrac{1}{2} a k'^3 r^2 + \ldots \tag{5}$$

where

$$k' = k - V'(0). \tag{6}$$

In this paper we neglect the electron mass and the excitation energy. Then the electron energy can be written as

$$k = |\boldsymbol{k}_1| = |\boldsymbol{k}_2|. \tag{7}$$

Using the Bohr–Mottelson model to describe the rotational states of the nucleus one obtains the following expression (Überall 1971) for the charge matrix element between initial and final states characterized by total angular momenta J and J' and projections M and M'

$$\langle J_{\mathrm{f}} M_{\mathrm{f}} | \rho(\boldsymbol{r}) | J_{\mathrm{i}} M_{\mathrm{i}} \rangle = (4\pi)^{1/2} (\hat{J}_{\mathrm{i}}/\hat{J}_{\mathrm{f}}) \sum_L \hat{L}^{-1} \rho_L(r) (J_{\mathrm{i}} J_{\mathrm{i}}, L0 | J_{\mathrm{f}} J_{\mathrm{i}}) \sum_M (J_{\mathrm{i}} M_{\mathrm{i}}, LM | J_{\mathrm{f}} M_{\mathrm{f}}) Y_{LM}(\hat{\boldsymbol{r}}) \tag{8}$$

where

$$\hat{J}_{\mathrm{i}} = (2J_{\mathrm{i}} + 1)^{1/2}. \tag{9}$$

We now choose the z axis to be along the direction of the momentum transfer $\boldsymbol{q} = \boldsymbol{k}_1 - \boldsymbol{k}_2$ and the x axis along $\boldsymbol{K}' = \boldsymbol{k}_1' + \boldsymbol{k}_2'$ (from which the azimuthal angle ϕ is to be measured) and make the required transformation for $Y_{LM}(\hat{\boldsymbol{r}})$:

$$Y_{LM}(\hat{\boldsymbol{r}}) \rightarrow \sum_{M''} D^L_{M''M}(0\beta 0) Y_{LM''}(\mu, \phi) \tag{10}$$

where

$$\cos\beta = \hat{\boldsymbol{q}} \cdot \hat{\boldsymbol{N}} \tag{11}$$

and

$$\mu = \hat{\boldsymbol{q}} \cdot \hat{\boldsymbol{r}}. \tag{12}$$

we have

$$\hat{\boldsymbol{K}}' \cdot \hat{\boldsymbol{r}} = (1-\mu^2)^{1/2} \cos\phi \tag{13}$$

and

$$Y_{LM}(\mu, \phi) = (-1)^M (\hat{L}/(4\pi)^{1/2}) [(L-|M|)!/(L+|M|)!]^{1/2} (1-\mu^2)^{M/2} \times (\mathrm{d}^M P_L(\mu)/(\mathrm{d}\mu)^M)\, \mathrm{e}^{\mathrm{i}M\phi}. \tag{14}$$

Following the procedure of I we obtain the scattering amplitude in the following form:

$$f_{J_i M_i \rightarrow J_f M_f} = \frac{2k}{q^2} \vartheta^+(\boldsymbol{k}_2)(\hat{J}_i/\hat{J}_f) \sum_L \sum_{M''} (J_i J_i, L0|J_f J_i)(J_i M_i, LM|J_f M_f)\hat{L}^2 \times D^L_{M''M}(0\beta 0) \mathscr{M}_{LM''} \vartheta(\boldsymbol{k}_1) \tag{15}$$

where

$$\mathscr{M}_{LM''} = 4\pi \hat{L}^{-2} \mathrm{i}^L \sum_{\epsilon = \pm 1} \int \rho_L(r) T_{LM''}(r, \mu = \epsilon) r^2 \,\mathrm{d}r \tag{16}$$

$$T_{LM''}(r, \mu) = -(-1)^L \frac{\mathrm{i}^L}{(4\pi)^{1/2}} \hat{L}^{-1} \int_0^{2\pi} \frac{F(\boldsymbol{r}) \exp(\mathrm{i}\boldsymbol{q}' \cdot \boldsymbol{r} + \mathrm{i}\Phi(\boldsymbol{r}))}{C^2(\boldsymbol{r})} Y_{LM''}(\mu, \phi)\, \mathrm{d}\mu\, \mathrm{d}\phi. \tag{17}$$

The quantities $(\mathrm{i}\boldsymbol{q}' \cdot \boldsymbol{r} + \mathrm{i}\Phi(\boldsymbol{r}))$, $C(\boldsymbol{r})$ and $F(\boldsymbol{r})$ occurring in (17) are defined by the following relations:

$$\begin{aligned} \boldsymbol{q}' \cdot \boldsymbol{r} + \Phi(\boldsymbol{r}) = {} & \boldsymbol{q}' \cdot \boldsymbol{r} - \tfrac{1}{2}a(k'r)^2 \boldsymbol{q}' \cdot \boldsymbol{r} + \tfrac{1}{12} a\boldsymbol{q}' \cdot \boldsymbol{r}[3(\boldsymbol{K}' \cdot \boldsymbol{r})^2 + (\boldsymbol{q}' \cdot \boldsymbol{r})^2] \\ & - \tfrac{1}{2}b[4(rk')^2 - (\boldsymbol{r} \cdot \boldsymbol{q}')^2 - (\boldsymbol{r} \cdot \boldsymbol{K}')^2] \\ & + \tfrac{1}{8}c[4(rk')^2 - (\boldsymbol{r} \cdot \boldsymbol{q}')^2 - (\boldsymbol{r} \cdot \boldsymbol{K}')^2]^2 + \tfrac{1}{2}c(\boldsymbol{r} \cdot \boldsymbol{q}')^2 (\boldsymbol{r} \cdot \boldsymbol{K}')^2 \end{aligned} \tag{18}$$

$$q'^2 C^2(\boldsymbol{r}) = |\nabla(\boldsymbol{q}' \cdot \boldsymbol{r} + \Phi(\boldsymbol{r}))|^2 \tag{19}$$

$$\begin{aligned} & \vartheta(\boldsymbol{k}_2)^\dagger F(\boldsymbol{r}) \vartheta(\boldsymbol{k}_1) \\ & \quad = \vartheta(\boldsymbol{k}_2)^\dagger \{1 + 3b\boldsymbol{q}' \cdot \boldsymbol{r} + a[(\boldsymbol{r} \cdot \boldsymbol{q}')^2 + (\boldsymbol{r} \cdot \boldsymbol{K}')^2 - 2(rk')^2] \\ & \qquad - \tfrac{5}{2}c\boldsymbol{q}' \cdot \boldsymbol{r}[4(k'r)^2 - (\boldsymbol{q}' \cdot \boldsymbol{r})^2 - 3(\boldsymbol{K}' \cdot \boldsymbol{r})^2] \\ & \qquad - [\tfrac{1}{2}a + 2c\boldsymbol{q}' \cdot \boldsymbol{r}]\boldsymbol{K}' \cdot \boldsymbol{r}\boldsymbol{\sigma} \cdot \boldsymbol{r}k'\} \vartheta(\boldsymbol{k}_1) \end{aligned} \tag{20}$$

now

$$\boldsymbol{\sigma} \cdot \hat{\boldsymbol{k}}_1 \vartheta(\boldsymbol{k}_1) = \vartheta(\boldsymbol{k}_1), \qquad \boldsymbol{\sigma} \cdot \hat{\boldsymbol{k}}_2 \vartheta(\boldsymbol{k}_2) = \vartheta(\boldsymbol{k}_2). \tag{21}$$

Let us study the nature of the integral in (17) for different values of M''. The relations (18) to (21) and (12) to (14) enable us to show that the integrand without the factor

$Y_{LM''}(\mu, \phi)$, involves only even powers of $\cos\phi$ and consequently the integral vanishes when $M'' = 1, 3, 5\ldots$ (odd numbers). If the μ integration in (17) is done by the method of integration by parts, terms of decreasing magnitude occur; the first term vanishes if $M'' \neq 0$ because $Y_{LM''}(\mu, \phi)$ as given by (14) becomes zero in the limit $\mu = \pm 1$. The next term, after the ϕ integration is done, is of the order of $(Z/137) \times 1/(q'R)^3$ if the main term of the result of integration for $M'' = 0$ is $O(1/q'R)$. Here Z is the atomic number and R is the average nuclear radius. So if $q'R \gg 1$, we can neglect the contribution of $T_{LM''}$ for $M'' \neq 0$ to the scattering amplitude as a good approximation. Petkov *et al* (1967b) also made this approximation. The above mentioned considerations enable us to rewrite equation (15) in the following form:

$$f_{J_iM_i \to J_fM_f} = \frac{2(4\pi)^{1/2}k}{q^2} \vartheta^{\dagger}(\boldsymbol{k}_2)\vartheta(\boldsymbol{k}_1)(\hat{J}_i/\hat{J}_f)$$
$$\times \sum_L (J_iJ_i, L0|J_fJ_i)(J_iM_i, LM|J_fM_f)\hat{L}Y_{LM}(-\beta)\mathscr{M}_L \tag{22}$$

where,

$$\mathscr{M}_L = \mathscr{M}_{L0} = 4\pi\hat{L}^{-2}\mathrm{i}^L \sum_{\epsilon=\pm 1} \int \rho_L(r)T_L(r, \mu = \epsilon)r^2\,\mathrm{d}r \tag{23}$$

$$T_L(r, \mu = \epsilon) = -\frac{1}{2}(-1)^L \mathrm{i}^L \sum_v (-1)^v \epsilon \left(\frac{F(r, \mu = \epsilon)\exp(\mathrm{i}qr'\epsilon + \mathrm{i}\phi(r, \mu = \epsilon))}{C^2(r, \mu = \epsilon)(\mathrm{i}q'r)^{v+1}D^{v+1}(r, \mu = \epsilon)} \right)$$
$$\times \left(\frac{\partial}{\partial\mu}\right)^v P_L(\mu)\bigg|_{\mu=\epsilon}. \tag{24}$$

Following the procedure of I and III we write:

$$\Phi(r, \mu = \epsilon) = -\epsilon a(\tfrac{1}{2}q'k'^2 - \tfrac{1}{12}q'^3)r^3 - \tfrac{1}{2}bK'^2r^2 + \tfrac{1}{8}cK'^4r^4 \tag{25}$$

$$F(r, \mu = \epsilon) = 1 + 3\epsilon bq'r + a(q'^2 - 2k'^2)r^2 - \tfrac{5}{2}\epsilon cq'K'^2r^3 \tag{26}$$

$$C(r, \mu = \epsilon) = 1 - a(\tfrac{3}{2}k'^2 - \tfrac{1}{4}q'^2)r^2 - \epsilon bK'^2r/q' + \epsilon cK'^4r^3/(2q') \tag{27}$$

$$D(r, \mu = \epsilon) = 1 - a(\tfrac{3}{2}k'^2 - \tfrac{1}{2}q'^2)r^2 + \epsilon b(\tfrac{3}{2}q'^2 - 2k'^2)r/q' - \epsilon c[k'^2 - \tfrac{5}{4}q'^2]K'^2r^3/q' \tag{28}$$

$$K'^2 = 4k'^2 - q'^2. \tag{29}$$

The term 'a' occurring in (18) to (20) and (25) to (28) is already defined by equation (5). The terms 'b' and 'c' occurring in the above mentioned equations are defined by the following relations:

$$b = (\pi Ze^2/k'^2)\int_0^\infty \rho_0(r)\,\mathrm{d}r \tag{30}$$

$$c = -(\pi Ze^2/8k'^4)\int_0^\infty (\mathrm{d}\rho_0/\mathrm{d}r)\left(\frac{1}{r}\right)\mathrm{d}r. \tag{31}$$

In the Born approximation limit (ie in the absence of the distortion of the electron wave), obtained by putting $Ze^2 = 0$ in the expression for $\phi_k(\boldsymbol{r})$ and $S(\boldsymbol{r})$ occurring in equation (1), we have

$$F(r, \epsilon) = C(r, \epsilon) = D(r, \epsilon) = 1 + \Phi(r, \epsilon)/q'r = 1 \tag{32}$$

and

$$\sum_{\epsilon=\pm 1} T_L(r,\epsilon) \rightarrow j_L(qr). \tag{33}$$

Considering the effect of all the multipole moments of the charge distribution we obtain from (22) in the usual way the following differential scattering cross section for electrons scattered from aligned nuclei:

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{align}} = \frac{e^4\cos^2\theta/2}{4k^2\sin^4\theta/2}(-1)^{J_i-J_f}\hat{J}_i^2\sum_L\sum_{L'}\hat{L}^2\hat{L}'^2\mathcal{M}_L^*\mathcal{M}_{L'}(J_iJ_i\,L\,0|J_fJ_i)$$
$$\times(J_iJ_i,L'0|J_fJ_i)\sum_I f_I(L\,0,L'\,0|I\,0)W(J_iJ_iLL';IJ_f)P_I(\hat{\boldsymbol{q}}\,.\,\hat{N}) \tag{34}$$

where

$$f_I = \sum_{M_i} P(M_i)(-1)^{J_i-M_i}(J_i\,M_i,J_i-M_i|I\,0). \tag{35}$$

In equation (35) $P(M_i)$ describes the initial occupation probability of various magnetic substates in a space-fixed coordinate system (defined by the direction $\hat{N}$ of the aligning magnetic field). We may note that the structure of the expression (34), obtained here in the eikonal approximation, is identical with that evaluated in the Born approximation by Inopin and Tishchenko (1960).

When the nucleus is completely oriented along the direction of the momentum transfer, ie $\hat{N}\|\boldsymbol{q}$ and $P(M_i)=\delta_{M_iJ}$, the expression (34) reduces to

$$\left(\frac{d\sigma}{d\Omega}\right)^{\hat{\boldsymbol{q}}\|\hat{N}} = \frac{e^4\cos^2\theta/2}{4k^2\sin^4\theta/2}\left(\frac{\hat{J}_i}{\hat{J}_f}\right)^2\sum_{L,L'}\hat{L}^2\hat{L}'^2\mathcal{M}_L^*\mathcal{M}_{L'}(J_iJ_i,L\,0|J_fJ_i)^2(J_iJ_i,L'\,0|J_fJ_i)^2. \tag{36}$$

We note that equation (36) unlike equation (34) does not involve the Racah coefficient W.

Now we confine ourselves to the contributions of the monopole ($L=0$) and quadrupole ($L=2$) charge distributions to $(d\sigma/d\Omega)^{\text{align}}$ and neglect the contributions of higher multipoles ($L>2$) and write

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{align}}_{J_i\rightarrow J_f} = \frac{e^4\cos^2\theta/2}{4k^2\sin^4\theta/2}\{|\mathcal{M}_0|^2\delta_{J_iJ_f}+2P_2(\hat{\boldsymbol{q}}\,.\,\hat{N})\,\text{Re}(\mathcal{M}_2^*\mathcal{M}_0)\hat{J}_i^2(J_iJ_i,J_i-J_i|2\,0)$$
$$\times f_2\delta_{J_iJ_f}+\hat{J}_f^2|\mathcal{M}_2|^2(J_iJ_i,J_f-J_i|2\,0)^2[1+5(-1)^{J_i-J_f}P_2(\hat{\boldsymbol{q}},\hat{N})\hat{J}_i^2$$
$$\times(2\,0,2\,0|2\,0)W(J_iJ_i\,2\,2;2\,J_f)f_2+5(-1)^{J_i-J_f}P_4(\hat{\boldsymbol{q}},\hat{N})\hat{J}_i^2(2\,0,2\,0|4\,0)$$
$$\times W(J_iJ_i2\,2;4\,J_f)f_4]\}. \tag{37}$$

2.1. Branching ratio:

For a nucleus fully oriented along the direction of the momentum transfer $\boldsymbol{q}$ we obtain from (36) and (37) the following expression for the branching ratio for excitations to two different rotational levels of the ground state rotational band, characterized by J_f and $J_{f'}$:

$$R^{\hat{N}\|\hat{\boldsymbol{q}}} = \frac{d\sigma^{\hat{N}\|\hat{\boldsymbol{q}}}_{J_i\rightarrow J_f}}{d\sigma^{\hat{N}\|\hat{\boldsymbol{q}}}_{J_i\rightarrow J_{f'}}} = \frac{(J_iJ_i,2\,0|J_fJ_i)^4\hat{J}_{f'}^2}{(J_iJ_i,2\,0|J_{f'}J_i)^4\hat{J}_f^2}. \tag{38}$$

The corresponding branching ratio for randomly oriented nuclei, $R^{\rm ran}$ (for which $f_I = \hat{J}_{\rm i}^{-1}\delta_{I_0}$) evaluated in the eikonal approximation is found to be identical to that derived by Inopin and Tishchenko (1960) in the Born approximation.

When $J_{\rm i} = \frac{7}{2}$, $J_{\rm f} = \frac{9}{2}$ and $J_{\rm f}' = \frac{11}{2}$ we get $R^{\hat{N}\parallel\hat{q}} = 18{\cdot}15$, $R^{\rm ran} = 3{\cdot}96$ and $R^{\hat{N}\perp\hat{q}} = 2{\cdot}694$, where $R^{\hat{N}\perp\hat{q}}$ implies that the nucleus is taken to be fully oriented normal to the scattering plane. For partially oriented nuclei we get the following branching ratio

$$\frac{(\mathrm{d}\sigma^{\rm align}_{J_{\rm i}\to J_{\rm f}} - \mathrm{d}\sigma^{\rm ran}_{J_{\rm i}\to J_{\rm f}})/\mathrm{d}\sigma^{\rm ran}_{J_{\rm i}\to J_{\rm f}}}{(\mathrm{d}\sigma^{\rm align}_{J_{\rm i}\to J_{\rm f}'} - \mathrm{d}\sigma^{\rm ran}_{J_{\rm i}\to J_{\rm f}'})/\mathrm{d}\sigma^{\rm ran}_{J_{\rm i}\to J_{\rm f}'}} = (-1)^{J_{\rm f}'-J_{\rm f}}$$

$$\times\frac{f_2(2\,0,2\,0|2\,0)W(J_{\rm i}J_{\rm i}2\,2;2J_{\rm f})P_2(\cos\beta)+f_4(2\,0,2\,0|4\,0)W(J_{\rm i}J_{\rm i}2\,2;4J_{\rm f})P_4(\cos\beta)}{f_2(2\,0,2\,0|2\,0)W(J_{\rm i}J_{\rm i}2\,2;2J_{\rm f'})P_2(\cos\beta)+f_4(2\,0,2\,0|4\,0)W(J_{\rm i}J_{\rm i}2\,2;4J_{\rm f'})P_4(\cos\beta)}. \quad (39)$$

Let

$$P_2(\cos\beta_2) = 0 \qquad \text{and} \qquad P_4(\cos\beta_4) = 0\ldots. \quad (40)$$

When $\beta(=\measuredangle(\hat{q},\hat{N}))$ is equal to either β_2 or β_4, the right-hand side of the relation (39) becomes independent of the degree of orientation characterized by f_2 and f_4. In the case $\beta = \beta_2(\beta_4)$ it is found from (39) and (37) that measurement of $\mathrm{d}\sigma^{\rm align}_{J_{\rm i}\to J_{\rm f}}/\mathrm{d}\sigma^{\rm ran}_{J_{\rm i}\to J_{\rm f}} - 1$ enables us to determine the Fano tensors $f_4(f_2)$.

Let us evaluate $\mathscr{M}_0$ and $\mathscr{M}_2$ defined by the integral in equation (23) for the following two forms for the charge distribution:

$$\text{Form 1:} \qquad \rho_0^W(r) = \rho_0(r) = \eta F(r); \qquad \rho_2^W(r) = \alpha^W r\frac{\mathrm{d}F(r)}{\mathrm{d}r} \quad (41)$$

$$\text{Form 2:} \qquad \rho_0^P(r) = \rho_0(r) = \eta F(r); \qquad \rho_2^P(r) = \alpha^P R\frac{\mathrm{d}F(r)}{\mathrm{d}r}. \quad (42)$$

The Fermi distribution $F(r)$ is given by

$$F(r) = 1/\{1+\exp[(r-R)/B]\} \quad (43)$$

where R is the half-density radius and B is the fall-off parameter. Form 1 and form 2 for the charge distribution were used by Wright (1969) and Petkov *et al* (1967a, b) respectively. In III the integral for $\mathscr{M}_0$ was expressed as the sum of an integral along the imaginary axis, an integral along contour $C^{(\epsilon)}$ of infinite radius and the sum of the residues at the poles $r_s^{(\epsilon)} = R+\mathrm{i}\epsilon(2s+1)\pi B(\epsilon = \pm 1, s = 0, 1, 2, \ldots)$ of the Fermi distribution $F(r)$ in the complex plane. The integral along the imaginary axis was found to be negligible if $R \gg B$. Petkov *et al* (1967a) further observed that the integral along $C^{(\epsilon)}$ vanishes if the following condition is fulfilled:

$$\mathrm{Re}\{\mathrm{i}\Phi(r\,\mathrm{e}^{\mathrm{i}\phi}, \mu = \epsilon) < qr[\epsilon\sin\phi+\cos\phi/(Bq)] \quad -\pi/2 \leqslant \phi \leqslant \pi/2, \qquad r\to\infty. \quad (44)$$

This condition is satisfied for the asymptotic phase

$$\Phi \sim C\ln r. \quad (45)$$

Following the above mentioned analytic method of III and using form 1 for the charge distribution we obtain

$$\mathscr{M}_0(q) = \eta\sum_{\epsilon=\pm 1}\ \sum_{s=0,1,2,\ldots} Y(r_s^{(\epsilon)}, \epsilon) \quad (46)$$

$$\mathcal{M}_2^W(q) = -\frac{\alpha^W}{5}\sum_{\epsilon=\pm 1}\sum_{s=0,1,2,\ldots} Y(r_s^{(\epsilon)},\epsilon)\left(\mathrm{i}\epsilon q' r_s^{(\epsilon)} C(r_s^{(\epsilon)},\epsilon) - \frac{3C(r_s^{(\epsilon)},\epsilon)}{D(r_s^{(\epsilon)},\epsilon)} + 2 - \frac{3\mathrm{i}\epsilon C(r_s^{(\epsilon)},\epsilon)}{q' r_s^{(\epsilon)} D^2(r_s^{(\epsilon)},\epsilon)} + \frac{3\mathrm{i}\epsilon}{q' r_s^{(\epsilon)} D(r_s^{(\epsilon)},\epsilon)}\right). \tag{47}$$

Similarly using form 2 for the charge distribution we obtain

$$\mathcal{M}_2^P(q) = -\frac{\alpha^P}{5}\sum_{\epsilon=\pm 1}\sum_{s=0,1,2,\ldots} Y(r_s^{(\epsilon)},\epsilon)\left(\mathrm{i}\epsilon q' R C(r_s^{(\epsilon)},\epsilon) + \frac{R}{r_s^{(\epsilon)}} - \frac{3R}{r_s^{(\epsilon)}}\frac{C(r_s^{(\epsilon)},\epsilon)}{D(r_s^{(\epsilon)},\epsilon)} + \frac{3R}{q'^2 (r_s^{(\epsilon)})^3 D^2(r_s^{(\epsilon)},\epsilon)} - \frac{3\mathrm{i}\epsilon C(r_s^{(\epsilon)},\epsilon) R}{q'(r_s^{(\epsilon)})^2 D^2(r_s^{(\epsilon)},\epsilon)}\right) \tag{48}$$

where,

$$Y(r_s^{(\epsilon)},\epsilon) = \frac{4\pi^2 B}{q'}\frac{F(r_s^{(\epsilon)},\mu=\epsilon) r_s^{(\epsilon)}}{D(r_s^{(\epsilon)},\mu=\epsilon) C^2(r_s^{(\epsilon)},\mu=\epsilon)}\exp[\mathrm{i}\epsilon q' r_s^{(\epsilon)} + \mathrm{i}\Phi(r_s^{(\epsilon)},\mu=\epsilon)] \tag{49}$$

$\mathcal{M}_0(q)$ is the same for both forms. It may be stated that equation (48) for $\mathcal{M}_2^P(q)$ is obtained by adding a correction term $r_s^{(\epsilon)}\,\mathrm{d}L/\mathrm{d}r$ to $[1+\mathrm{i}\epsilon q' r_s^{(\epsilon)} C(r_s^{(\epsilon)},\epsilon)]\ L$ occurring in Petkov *et al* (1967b) where

$$L = 1/D(r_s^{(\epsilon)},\epsilon) + 3\mathrm{i}\epsilon/(q' r_s^{(\epsilon)} D^2(r_s^{(\epsilon)},\epsilon)) - 3/((q' r_s^{(\epsilon)})^2 D^3(r_s^{(\epsilon)})).$$

The above correction term leads to quantities which are of the order $1/(qR)^3$ and $1/(qR)^4$ if the main term is of the order of $1/(qR)$. Now in the eikonal approximation which involves an asymptotic series in $1/(qR)$ Yennie (1965) retained the first term which is $O(1/qR)$ and neglected the next term of the series which is $O(Ze^2/(qR)^3)$. For light nuclei we may not neglect a quantity of the order of $1/(qR)^3$ (in contrast with the quantity $O(Ze^2/(qR)^3)$) which comes from the term $r_s^{(\epsilon)}\,\mathrm{d}L/\mathrm{d}r$ neglected in II. Further, the retention of the above correction term yields the correct scattering amplitude in the Born approximation limit given by (32). We have

$$\mathcal{M}_2^{W(\mathrm{Born})} = \frac{4\pi^2\alpha^W B}{5q}\{[qR^2 - \pi B\,\mathrm{cotanh}(\pi Bq) - 2q(\pi B\,\mathrm{cotanh}\,(\pi Bq))^2 + (\pi B)^2 q] \times \sin(qR) + R(1+2\pi Bq\,\mathrm{cotanh}\,(\pi Bq))\cos(qR)\}\,\mathrm{cosech}(\pi Bq) \tag{50}$$

and

$$\mathcal{M}_2^{P(\mathrm{Born})} = \frac{4\pi^2\alpha^P BR}{5q}\left(qR\,\mathrm{cosech}(\pi Bq)\sin(qR) + (2+\pi Bq\,\mathrm{cotanh}(\pi Bq)) \times \mathrm{cosech}(\pi Bq)\cos(qR) - \frac{3}{q^2}\int_0^q q''\,\mathrm{cosech}(\pi Bq'')\cos(q''R)\,\mathrm{d}q'' + \frac{3}{4(Bq)^2}\mathrm{sech}^2(R/2B)\right). \tag{51}$$

The expression for $\mathcal{M}_0^{\mathrm{Born}}$ was already given in III. Setting the fall-off parameter $B = 0$ in (50) and (51) we obtain the corresponding equation for $\mathcal{M}_2^{(\mathrm{Born})}$ given by Greenstein (1966) for a δ function quadrupole distribution.

Using the following relation for the quadrupole moment Q:

$$Q = \frac{8\pi}{5}\int_0^\infty r^4\rho_2(r)\,\mathrm{d}r \tag{52}$$

and writing $d = B/R$ we give below the formulae for α^W and α^P defined by (41) and (42)

$$\alpha^W = -\frac{5Q}{8\pi R^5}\left[1+\frac{10}{3}(\pi d)^2+\frac{7}{3}(\pi d)^4+120d^5\left(e^{-1/d}-\frac{e^{-2/d}}{2^5}+\frac{e^{-3/d}}{3^5}\right)\right]^{-1} \tag{53}$$

$$\alpha^P = -\frac{5Q}{8\pi R^5}\left[1+2(\pi d)^2+\frac{7(\pi d)^4}{15}-24d^4\left(e^{-1/d}-\frac{e^{-2/d}}{2^4}+\frac{e^{-3/d}}{3^4}\right)\right]^{-1}. \tag{54}$$

From (31) we obtain the following expression for c:

$$c \simeq \tfrac{3}{32}(Ze^2/(k'R)^4)(1-\tfrac{2}{3}\pi^2 d^2) \tag{55}$$

which differs from that for 'c' given in III. This, however, unlike 'c' of Petkov *et al* (1967a) is found to agree (which it should) with that given by Yennie (1965) in the limit of zero value of the fall-off parameter B. The expressions for a, b and η defined by (5), (30) and (41) respectively are the same as given in III.

3. Calculation

As a special application of the relation (37) we investigate the scattering of high energy electrons from aligned ^{165}Ho nuclei ($Z = 67$, spin $= \frac{7}{2}$). Summing the expression given by (37) over all the final states of the rotational band we get

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{align}} = \sum_{J_f = J_i, J_i+1, J_i+2}\left(\frac{d\sigma}{d\Omega}\right)^{\text{align}}_{J_i\to J_f}. \tag{56}$$

For a randomly-oriented nucleus we have

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{ran}} = \frac{e^4}{4k^2}(|\mathcal{M}_0|^2+5|\mathcal{M}_2|^2). \tag{57}$$

We now define a quantity Δ giving the alignment effect as

$$\Delta = (d\sigma^{\text{align}}/d\sigma^{\text{ran}})-1. \tag{58}$$

From (37), (56) and (57) it is evident that the quadrupole transition probability is proportional to the following quantities:

$$W_1 = \left(\frac{d\sigma}{d\Omega}\right)^{\text{align}}(\text{for } \hat{\boldsymbol{N}}.\hat{\boldsymbol{q}} = 1/\sqrt{3}) - \left(\frac{d\sigma}{d\Omega}\right)^{\text{ran}} \tag{59}$$

and

$$W_2 = \left(\frac{d\sigma}{d\Omega}\right)^{\text{align}}(\text{for } \hat{\boldsymbol{N}}\|\hat{\boldsymbol{q}}) + 2\left(\frac{d\sigma}{d\Omega}\right)^{\text{align}}(\text{for } \hat{\boldsymbol{N}}\|(\boldsymbol{k}_1\times\boldsymbol{k}_2)) - 3\left(\frac{d\sigma}{d\Omega}\right)^{\text{ran}}. \tag{60}$$

Here we can make an estimate of the amount by which the results for $(d\sigma/d\Omega)^{\text{ran}}$ and Δ (both calculated by Wright's method) change if one uses form 2 instead of form 1 for $\rho_2(r)$. This can be done from the knowledge of the corresponding results obtained by the method of eikonal approximation and from the fact that the two methods yield results in close agreement with each other. Thus we can write

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{Ph}}_2 \simeq \left(\frac{d\sigma}{d\Omega}\right)^{\text{Ph}}_1 + \left(\frac{d\sigma}{d\Omega}\right)^{\text{Ei}}_2 - \left(\frac{d\sigma}{d\Omega}\right)^{\text{Ei}}_1 \tag{61}$$

where the superscripts 'Ph' and 'Ei' refer to Wright's (1969) method and the eikonal method and the subscripts 1 and 2 refer to forms 1 and 2 given by (41) and (42) respectively. The relation (61) holds for both aligned and non-aligned nuclei. Using the relation (61) and (58) we can obtain an appropriate approximate relation for the alignment effect Δ.

Petkov *et al* (1967a) observed that only the $s = 0$ term of the series in (46), (47) and (48) gives the main contribution, each succeeding term of the series being smaller than the preceding term by a factor of the order of $\exp(2\pi qB)$. Now some precaution is needed in evaluating the above mentioned series for the following reason. It is found that this series for the scattering amplitude first converges and then it begins to diverge when s (which can take the values 0, 1, 2, 3, ...) exceeds a certain large value (depending upon q). This is expected because the function $\Phi(r_s^{(\epsilon)}, \epsilon)$ (unlike the asymptotic form of the function given by (45)) occurring in (49) may not satisfy the relation (44) for very large values of s. This can be explained by the fact that $\Phi(r_s^{(\epsilon)}, \epsilon)$ occurring in (25) and (49) was obtained in I by an analytic expansion about $r = 0$ to match the assumed expansion of the potential given by (5) and may not be used when s is very large and the corresponding pole $r_s^{(\epsilon)}$ is located far from the origin. In this connection we may point out that the calculation performed for high energy ($k = 300$ MeV) electron scattering from the Bi ($Z = 83$) nucleus (investigated in III) shows that $(\mathrm{d}\sigma/\mathrm{d}\Omega)$ diverges if $s > 5$ (8) when the scattering angle is 30° (40°). It is also found in the above case that the contribution of the term corresponding to $s > 1$ (s should not be, however, very large) of the series to the value of $\mathrm{d}\sigma/\mathrm{d}\Omega$ is very small. In our calculation for high energy ($k = 200$ MeV) electron scattering from ^{165}Ho ($Z = 67$) we have considered the terms corresponding to $s = 0$

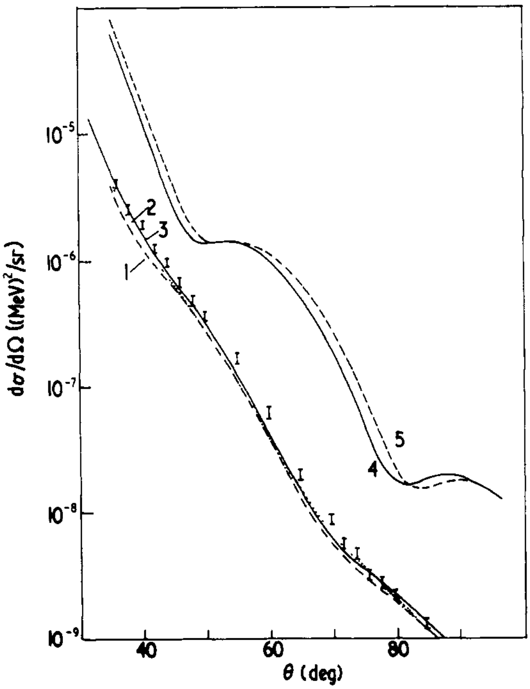

Figure 1. Curve 1(2) represents $(\mathrm{d}\sigma/\mathrm{d}\Omega)^{\mathrm{ran}}$ for electron scattering from ^{165}Ho using form 1(2) for the charge distribution. Curve 4(5) represents $100W_1$ (related to quadrupole charge) for form 1(2). The eikonal approximation is used for all the curves except curve 3 which represents Wright's result for form 2. Experimental points are from Safrata *et al* (1967).

and $s = 1$ of the series in (46), (47) and (48). It may be noted that the scattering of 250 MeV electrons from gold ($Z = 83$), treated by Yennie by the eikonal method (which is reliable for $qR \gg 1$) gives a good description for scattering angle $\theta > 40°$. In view of that, the same method is expected also to give reliable result for $\theta > 40°$ when applied to 200 MeV electron scattering from ^{165}Ho ($Z = 67$).

We take the following values for the quadrupole moment Q, $P(M_i)$, and the Fermi charge density parameters used by Wright (1969):

$Q = 8\,b$, $R = 6{\cdot}12$ fm, $B = 0{\cdot}65$ fm

$P(\frac{7}{2}) = 0{\cdot}561$, $P(\frac{5}{2}) = 0{\cdot}254$, $P(\frac{3}{2}) = 0{\cdot}110$, $P(\frac{1}{2}) = 0{\cdot}046$

$P(-\frac{1}{2}) = 0{\cdot}018$, $P(-\frac{3}{2}) = 0{\cdot}007$, $P(-\frac{5}{2}) = 0{\cdot}003$, $P(-{}^{7/2}) = 0{\cdot}001$.

The numerical calculations are displayed in figures 1–4.

In figure 1, which also shows the experimental result of Safrata *et al* (1967), curve 1(2) represents $(d\sigma/d\Omega)^{ran}$ against θ calculated in the eikonal approximation assuming form 1(2) for the charge distribution. Curve 3 is Wright's (1969) result. Curve 4(5), corresponding to form 1(2), represents $100 \times W_1$, (defined by (59)) against θ for fully oriented nuclei.

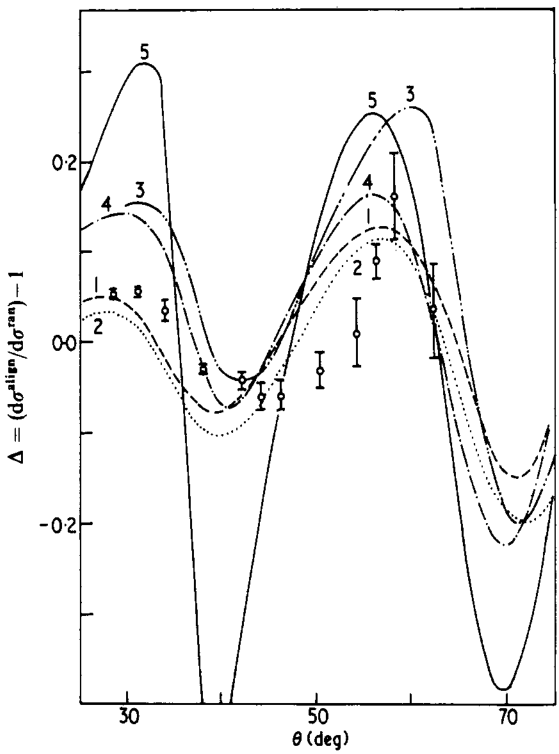

Figure 2. Curve 1(2) represents $\Delta = (d\sigma^{align}/d\sigma^{ran}) - 1$ against θ (calculated in the eikonal approximation) for electron scattering from ^{165}Ho using form 1(2). Curve 3 represents Wright's result for form 1. Curve 4(5) represents Δ evaluated in the eikonal (Schiff–Tiemann) approximation using a δ function quadrupole distribution. Experimental points are from Safrata *et al* (1967).

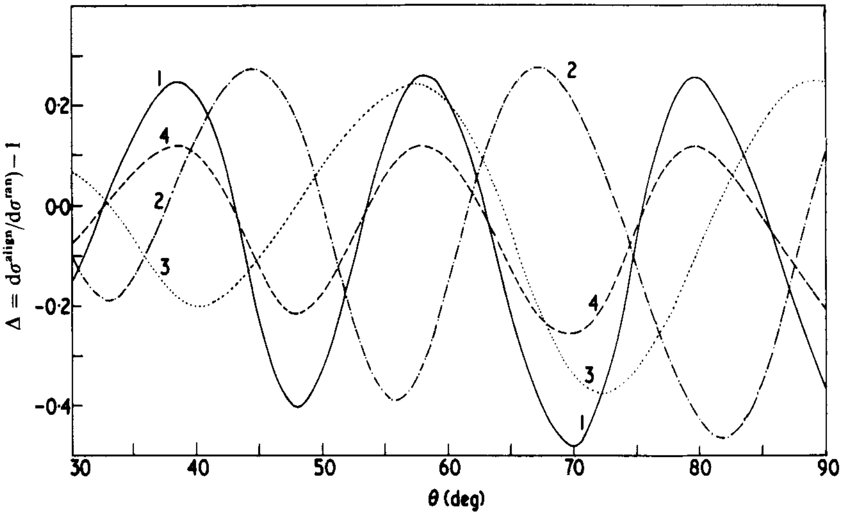

Figure 3. Curves 1, 2 and 3 represent $\Delta = (d\sigma^{align}/d\sigma^{ran})-1$ against θ for 300 MeV electrons, 300 MeV positron and 200 MeV electron scattering from ^{165}Ho fully-oriented normal to the scattering plane. Curve 4 gives Δ for 300 MeV electron scattering from partially-oriented ^{165}Ho.

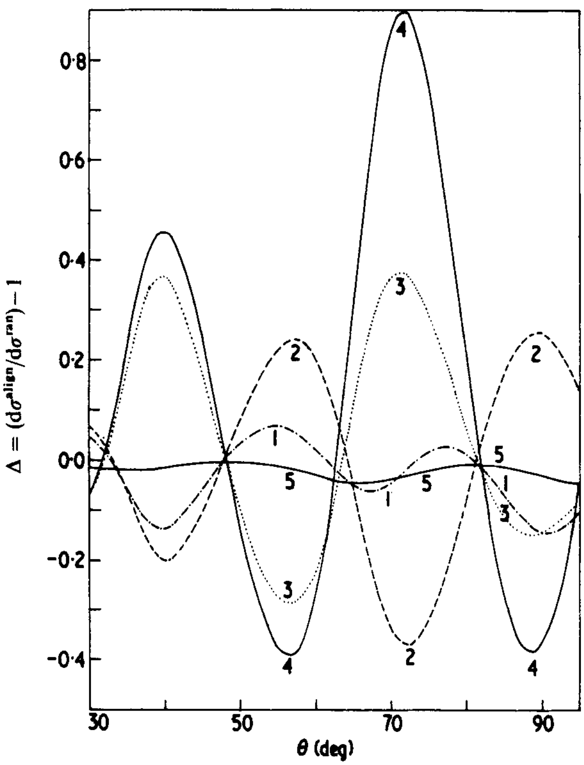

Figure 4. Plot of $(d\sigma^{align}/d\sigma^{ran})-1$ against θ for 200 MeV electron scattering by ^{165}Ho (fully aligned) which may be oriented in different directions. Orientation levels are given in the text.

In figure 2 are shown experimental and theoretical values for Δ against θ. Curve 1(2) is obtained from the eikonal approximation using form 1(2). Δ, calculated by Wright's method using form 1, is represented by curve 3. Curves 4 and 5 represent the calculations carried out in the eikonal approximation and the Schiff–Tiemann approximation (Greenstein 1966) respectively, using $Q = 8{\cdot}56$ b, $R = 6.58$ fm, the step-function charge distribution for the monopole part and a δ function quadrupole distribution.

In figure 3 curves 1, 2 and 3 represent Δ plotted against θ for 300 MeV electron, 300 MeV positron and 200 MeV electron scattering respectively when the nucleus is taken to be fully oriented. Curve 4 refers to 300 MeV electron scattering from a partially oriented nucleus using the values of $P(M_i)$ given before. In figure 2 and figure 3 the nuclear orientation $\hat{N}$ is always taken to be normal to the scattering plane.

In figure 4 we have shown the variation of Δ with θ for the case of fully-oriented nuclei and 200 MeV electrons by curves 1, 2, 3 and 4 corresponding to the orientations $(\hat{N}\|\boldsymbol{k}_1)$, $(\hat{N}\|\boldsymbol{k}_1 \times \boldsymbol{k}_2)$, $(\hat{N}\|(\boldsymbol{k}_1 \times \boldsymbol{k}_2) \times \boldsymbol{k}_1)$ and $(\hat{N}\|\boldsymbol{q})$ respectively. Curve 5 represents the same when $\hat{N} \cdot \hat{\boldsymbol{q}} = \cos\beta_2 = 1\sqrt{3}$ (see (40)). Form 2 for the nuclear charge distribution is used to calculate Δ in both figure 3 and figure 4.

4. Discussion

It is found from figure 1 that the use of the eikonal approximation to evaluate $(\mathrm{d}\sigma/\mathrm{d}\Omega)^{\mathrm{ran}}$ yields a result which is in close agreement with that obtained by Wright (1969) and also with the experimental data. The quantity W_1 (defined by (59)), represented by curves 4 and 5 in figure 1, may provide us with information about the inelastic scattering (due to a quadrupole transition charge density) which cannot be separated experimentally from the elastic scattering for ^{165}Ho. Figure 2 shows that Wright's (1969) result for Δ (calculated for form 1) is too large compared to the experimental result of Safrata *et al* (1967). The eikonal approximation (which in the present case is reliable for $\theta > 40°$) with either form 1 or form 2 leads to a result for Δ which is lower than that given by Wright (1969) and is in better accord with the experimental finding. This observation, for the particular case considered in this paper, need not be generalized. Now Wright (1969) pointed out that there was the possibility that the experimental alignment parameters were not correctly known. In such a case it is not possible to compare the results obtained by the two methods with the experimental data. In the eikonal approximation the assumption of form 2 instead of form 1 leads to a little better fit with the experimental data for Δ. The approximate relation (61) for $(\mathrm{d}\sigma/\mathrm{d}\Omega)$, a similar one for Δ (obtained from (61) and (58)) and the relative shifts between the curves in figure 1 and figure 2 show that for large-angle scattering we may expect somewhat better agreement with the experimental value for $(\mathrm{d}\sigma/\mathrm{d}\Omega)$ and Δ if we employ form 2 instead of form 1 in the phase-shift calculation of Wright (1969).

Curves 4 and 5 in figure 2 show that when we assume a δ function quadrupole distribution, the quantity Δ, evaluated in the eikonal approximation, is in better agreement with the experimental value than that calculated in the Schiff–Tiemann approximation.

It is evident from curves 1 and 3 of figure 3 that the gap between the successive maxima of Δ is narrowed with the increase in energy. A study of curves 1 and 2 shows that the above mentioned gap is smaller for electrons than that for positrons. Curves 1 and 4 show that there is little change in the position of the maxima and the minima of Δ with different degrees of alignment. Wright (1969) found that the alignment effect defined by Δ was larger for the case $\hat{N}\|\boldsymbol{k}_1 \times (\boldsymbol{k}_1 \times \boldsymbol{k}_2)$ than for the case $\hat{N}\|(\boldsymbol{k}_1 \times \boldsymbol{k}_2)$. A study of figure 4 reveals that a still larger effect of alignment is obtained for the case $\hat{N}\|\boldsymbol{q}$ (represented by curve 4). It may be seen that Δ is very small for $\hat{\boldsymbol{N}}\cdot\hat{\boldsymbol{q}} = 1/\sqrt{3}$. Curves 4 and 2 also show that Δ (for $\hat{N}\|\boldsymbol{q}$) is approximately two times Δ (for $\hat{N}\|(\boldsymbol{k}_1 \times \boldsymbol{k}_2)$) in magnitude and opposite in sign. The above mentioned properties of Δ can be explained from the relations (58), (57), (56) and (37) and from the fact that the term involving f_4 is small. The same relations also show that Δ for $\hat{\boldsymbol{N}}\cdot\hat{\boldsymbol{q}} = 1/\sqrt{3}$ vanishes when the nuclear spin $I \leqslant \frac{3}{2}$. If we want to study Δ against $|\boldsymbol{q}|$ in the case $\hat{N}\|\boldsymbol{q}$, two different experimental methods can be adopted to alter $|\boldsymbol{q}|$ in a continuous manner, keeping the direction of $\hat{N}$ always parallel to $\boldsymbol{q}$. The first method consists of varying the scattering angle and $\hat{N}$, ie the direction of the aligning magnetic field, simultaneously in a suitable manner. In the second method we have to alter the energy of the electron beam, keeping the scattering angle θ fixed, so that the direction of the aligning magnetic field need not be changed. The second method is more suitable for experimental purposes. In conclusion we may say that more accurate experiments for different alignments and projectile energies in future will help us to know more about the structure of the deformed nucleus.

References

Glauber R J 1959 *Lectures in Theoretical Physics* vol 1 (New York: Interscience)
Greenstein H B 1966 *Phys. Rev.* **144** B 902–17
Inopin E V and Tishchenko B I 1960 *Sov. Phys.–JETP* **37** 932–8
Langworthy J and Überall H 1970 *Phys. Rev.* C **2** 911–9
Onley D S *et al* 1964 *Phys. Rev.* B **134** 945–52
Petkov I Zh *et al* 1967a *Sov. J. Nucl. Phys.* **4** 41–6
Petkov I Zh *et al* 1967b *Sov. J. Nucl. Phys.* **4** 395–9
Safrata R S *et al* 1967 *Phys. Rev. Lett.* **18** 667–70
Überall H 1971 *Electron Scattering from Complex Nuclei, Part A* (New York: Academic Press)
Wallace S J 1973 *Ann. Phys., NY* **78** 190–257
Wright L E 1969 *Nucl. Phys.* A **135** 139–44
Yennie D R 1965 *Phys. Rev.* **137** B882–903